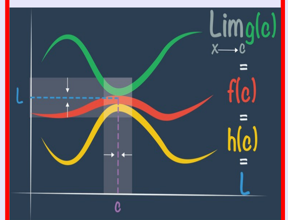
Calculus I

Lecture 2



Class QZ 1 Box Your Final Answer.

1) Solve $3(x-1)-5(x+2)=-13$

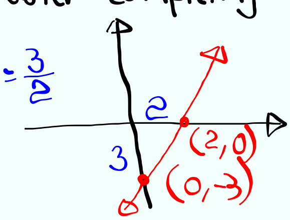
$3x-3-5x-10=-13$

$-2x-13=-13$ $\quad -2x=0$ $\quad \boxed{x=0}$ $\quad \{0\}$

2) Graph $3x-2y=6$ after completing the chart below:

x	y
0	-3
2	0

$m=\frac{3}{2}$



$3x-2y=6$

$-2y=-3x+6$

$\frac{-2}{-2}y=\frac{-3}{-2}x+\frac{6}{-2}$

$\boxed{y=\frac{3}{2}x-3}$

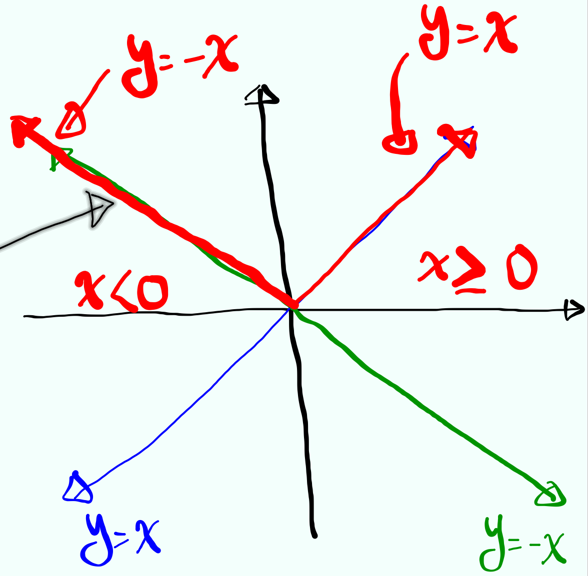
$y=mx+b$

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

$y = x$

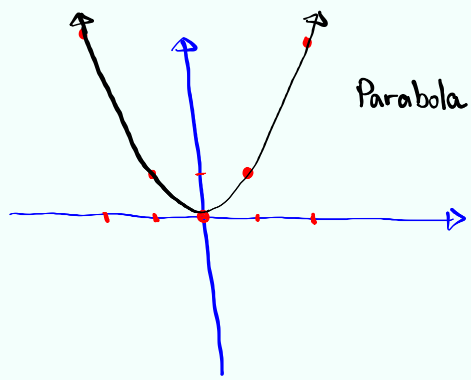
$y = -x$

$y = |x|$



$y = x^2$

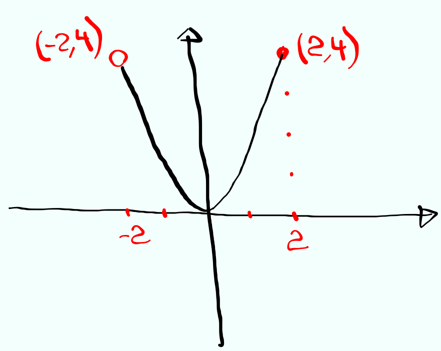
x	y
0	0
±1	1
±2	4
±3	9



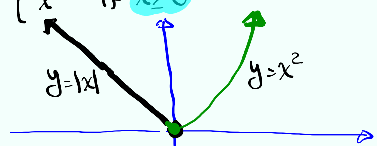
Graph $y = x^2$ for $-2 < x \leq 2$

Domain $(-2, 2]$

Range $[0, 4]$

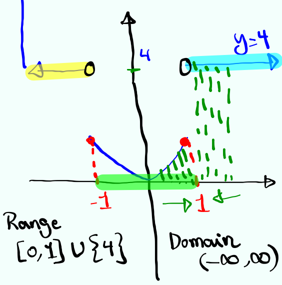


Graph $y = \begin{cases} |x| & \text{if } x<0 \\ x^2 & \text{if } x \geq 0 \end{cases}$ Piece-Wise



Graph

$y = \begin{cases} 4 & \text{if } x<-1 \\ x^2 & \text{if } -1 \leq x \leq 1 \\ 4 & \text{if } x>1 \end{cases}$



$y=4$ horizontal line

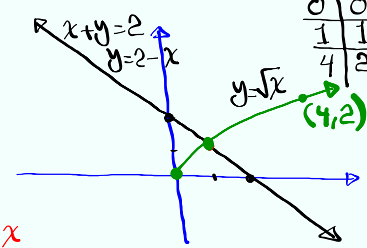
$y \to ?^{4}$ when $x \to 1^{+}$ from right

$y \to ?^{1}$ when $x \to 1^{-}$ from left

$\lim_{x \to 1^{+}} y = 4$ $\lim_{x \to 1^{-}} y = 1$

Graph $x+y=2$ and $y=\sqrt{x}$ in the Same Coordinate System. $x \geq 0$

x	y
0	0
1	1
4	2



$x+y=2$

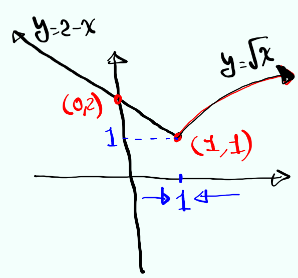
x	y
0	2
2	0
1	1

$y=2-x$

$x+y=2$

Graph

$y = \begin{cases} 2-x & \text{if } x \leq 1 \\ \sqrt{x} & \text{if } x>1 \end{cases}$



$\lim_{x \to 1^{+}} y = 1$

$\lim_{x \to 1^{-}} y = 1$

Unit Circle

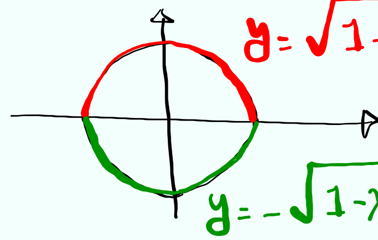
Center at $(0,0)$

Radius 1

$\Rightarrow x^2+y^2=1$

$y^2=1-x^2$

$y=\pm\sqrt{1-x^2}$



$y=\sqrt{1-x^2}$

$y=-\sqrt{1-x^2}$

Isolate x and graph your answers.

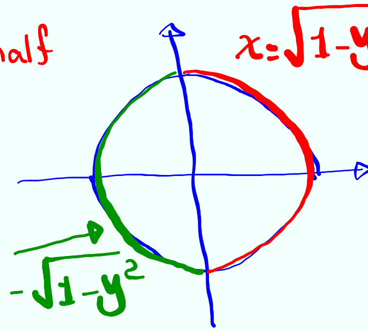
$x^2+y^2=1$

$x^2=1-y^2$

$x=\pm\sqrt{1-y^2}$

Right-half

left-half



$x=\sqrt{1-y^2}$

$x=-\sqrt{1-y^2}$

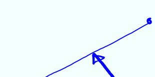
what is a circle?

It is a set of all points (x,y) that are same distance from a fixed point (h,k).

Radius

Center

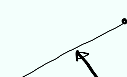
Distance formula



(x_2,y_2)

(x_1,y_1)

$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$



(x,y)

(h,k)

$R=\sqrt{(x-h)^2+(y-k)^2}$

$$\boxed{R^2=(x-h)^2+(y-k)^2}$$

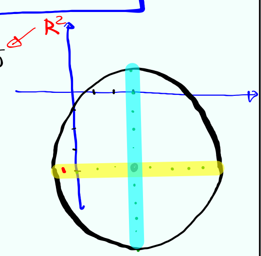
$(x-3)^2+(y+4)^2=25 \leftarrow R^2$

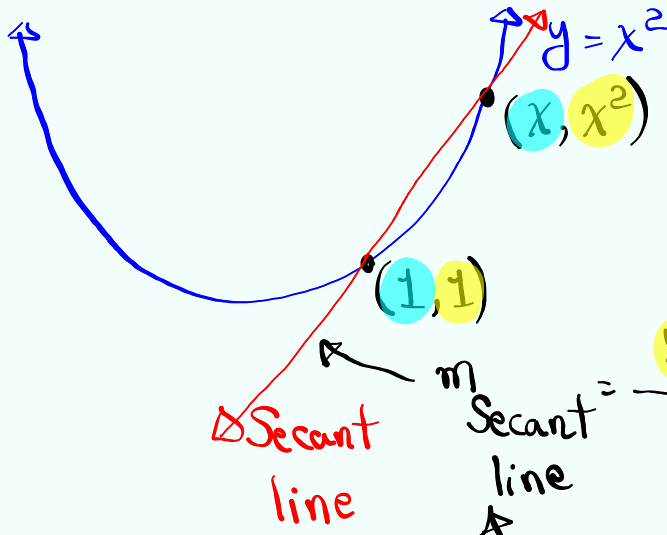
Center $(h,k)=(3,-4)$

Radius $R=5$

Domain $\rightarrow -2\le x\le 8$

Range $\rightarrow -9\le y\le 1$



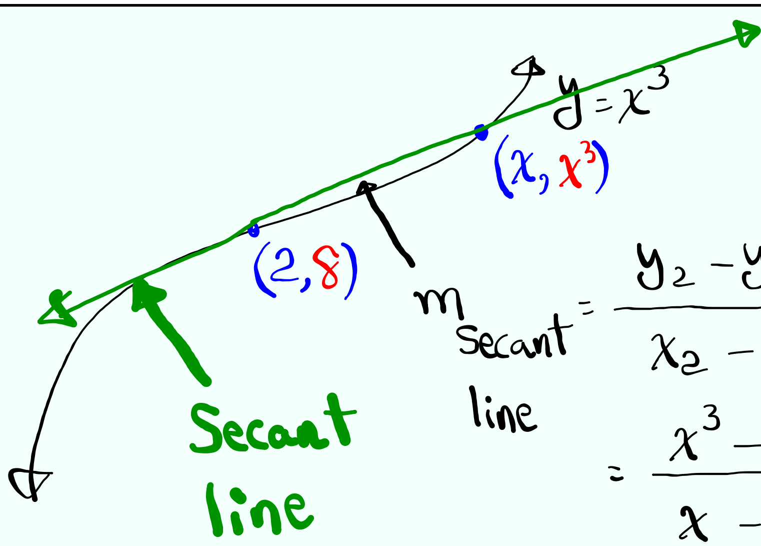


$A^2 - B^2$

$$m_{\text{secant line}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{x^2 - 1}{x - 1}$$

$$= \frac{(x+1)\cancel{(x-1)}}{\cancel{x-1}}$$

$$= \boxed{x+1}$$



$$m_{\text{secant line}} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{x^3 - 8}{x - 2}$$

$$= \frac{\cancel{(x-2)}(x^2 + 2x + 4)}{\cancel{x-2}}$$

$$= \boxed{x^2 + 2x + 4}$$

Hint:

$A^3 - B^3 = (A - B)(A^2 + AB + B^2)$

$x^3 - 2^3$

Class Quiz 2

Box Your Final Ans.

Simplify

$$\frac{x^2-12x+36}{x^2-36} = \frac{(x-6)\cancel{(x-6)}}{(x+6)\cancel{(x-6)}}$$

$$= \boxed{\frac{x-6}{x+6}}$$

$f(x)$

Function notation

f of x

For every x value, there is only one y-value

$f(x) = x^2$

$f(0) = 0^2 = 0$

$f(2) = 2^2 = 4$

$f(-3) = (-3)^2 = 9$

$$f(x+h) = (x+h)^2 = (x+h)(x+h)$$

$$= x^2 + xh + hx + h^2$$

$$= x^2 + 2hx + h^2$$

$f(x) = x^2 - 2x$

1) $f(0) = 0^2 - 2(0)$
$= \boxed{0}$

2) $f(2) = 2^2 - 2(2)$
$= 4 - 4 = \boxed{0}$

3) $f(-3) = (-3)^2 - 2(-3)$
$= 9 + 6$
$= \boxed{15}$

4) Simplify $f(x+h) - f(x)$

$= (x+h)^2 - 2(x+h) - (x^2 - 2x)$

$= (x+h)(x+h) - 2x - 2h - x^2 + 2x$

$= \cancel{x^2} + xh + hx + h^2 \cancel{-2x} - 2h \cancel{-x^2} \cancel{+2x}$

$= \boxed{2xh + h^2 - 2h} = h(2x + h - 2)$

Difference Quotient

$$\frac{f(x+h) - f(x)}{h}$$

ex: $f(x) = 2x + 5$

$$\frac{f(x+h) - f(x)}{h} = \frac{2(x+h) + 5 - (2x+5)}{h}$$

$$= \frac{\cancel{2x} + 2h \cancel{+5} \cancel{-2x} \cancel{-5}}{h} = \frac{2\cancel{h}}{\cancel{h}} = \boxed{2}$$

Simplify the difference quotient for $f(x)=x^2-4x+6$, then evaluate for $\boxed{h=0.}$

$$\frac{f(x+h)-f(x)}{h}=\frac{(x+h)^2-4(x+h)+6-(x^2-4x+6)}{h}$$

$$=\frac{(x+h)(x+h)-4x-4h+6-x^2+4x-6}{h}$$

$$=\frac{\cancel{x^2}+2xh+h^2-\cancel{4x}-4h\cancel{+6}\cancel{-x^2}\cancel{+4x}\cancel{-6}}{h}$$

$$=\frac{2xh+h^2-4h}{h}$$

$$=\frac{\cancel{h}(2x+h-4)}{\cancel{h}}=2x+h-4$$

for $h=0 \rightarrow \boxed{2x-4}$

Find the difference quotient for $f(x)=\frac{1}{x}$, Simplify, then evaluate for $h=0$.

$$\frac{f(x+h)-f(x)}{h}=\frac{\frac{1}{x+h}-\frac{1}{x}}{h}$$

$$=\frac{\cancel{(x+h)}\cdot x\cdot\frac{1}{\cancel{x+h}}-(x+h)\cdot\cancel{x}\cdot\frac{1}{\cancel{x}}}{(x+h)\cdot x\cdot h}$$

To simplify we multiply top and bottom by the LCD

LCD $=(x+h)x$

$$=\frac{x-(x+h)}{(x+h)\cdot x\cdot h}=\frac{-\cancel{h}^{\,1}}{(x+h)\cdot x\cdot\cancel{h}}$$

$$=\frac{-1}{(x+h)\cdot x}$$

For $h=0 \rightarrow \frac{-1}{(x+0)\cdot x}=\boxed{\frac{-1}{x^2}}$

Simplify

$$(\sin x+\cos x)^2+(\sin x-\cos x)^2$$

$$=(\sin x+\cos x)(\sin x+\cos x)+(\sin x-\cos x)(\sin x-\cos x)$$

$$=\sin^2 x\cancel{+\sin x\cos x}+\cancel{\cos x\sin x}+\cos^2 x$$

$$+\sin^2 x\cancel{-\sin x\cos x}\cancel{-\cos x\sin x}+\cos^2 x$$

$$=2\sin^2 x+2\cos^2 x=2(\underbrace{\sin^2 x+\cos^2 x}_{1})=2\cdot 1$$

$$=\boxed{2}$$